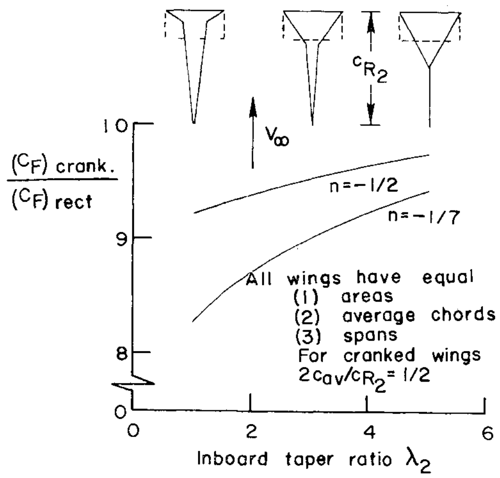

Fig 2 Ratios of the average skin-friction coefficient for cranked wings to rectangular wings

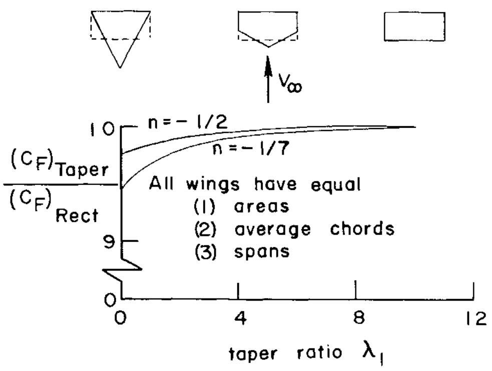

Fig 3 Ratios of average skin-friction coefficient for tapered wings to rectangular wings

for the limit $0 \leq \lambda_1 \leq 1$ where again

$$\lim_{\lambda_1 \to 1}\left[\frac{1 - (\lambda_1)^{n+2}}{1 - \lambda_1}\right] = n + 2$$

Effects of taper ratio variations on the foregoing skin-friction ratio are shown in Fig 3 The results indicate that for wings having straight leading edges and small taper ratios the skin friction cannot be obtained accurately from a Reynolds number based on its average chord

Reference

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Comment on "Velocity Defect Law for a Transpired Turbulent Boundary Layer"

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In their recent contribution to this Journal, Mickley and Smith[1] explain the behavior of transpired turbulent boundary layers in a novel way Former work of Mickley[2] was based on Rubesin's[3] application of the mixture-length theory to turbulent boundary layers with suction or injection

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This theory predicts a bilogarithmic mean velocity profile In contrast to this, a semilogarithmic mean velocity profile is characteristic of Clauser's[4] phenomelogical description of turbulent boundary layers It appears that Mickley and Smith abandoned the former theory in favor of the latter Their changed approach agrees with the development at our laboratory The results of a rather large number of experiments forced us to accept that turbulent boundary layers with distributed suction have semilogarithmic mean velocity profiles, without any positive tendency towards conformity with the bilogarithmic law The slope of the logarithmic part of the mean velocity profile appears to depend on the ratio v_0/u_τ only, as Fig 1 shows

The data, which exhibit rather much scatter because of the inaccuracy of determining the slope from experimental curves, suggest the following relation:

$$w^* = x_2(\partial U_1/\partial x_2) = 2\ 3\ u_\tau(1 + 9v_0/u_\tau) \qquad (1)$$

It is proposed to call w^* the "logarithmic velocity scale" The relation suggested by Mickley and Smith, i e , that the logarithmic velocity scale (equivalent to U_T^* in their notation) is proportional to the square root of the maximum Reynolds stress within the boundary layer, cannot be applied to aspired turbulent boundary layers For these layers the shear stress attains its maximum in a sharp peak at the wall, which is not representative for the level of Reynolds stress at the outer edge of the inner layer, as experiments performed by Favre et al [5] have shown

The empirical relation (1), however, is not very well suited to describe the behavior of turbulent boundary layers with moderate suction ($0\ 04 < -v_0/u_\tau < 0\ 10$) These layers have a relatively thick viscous sublayer which is described by

$$\frac{v_0 U_1}{u_\tau^2} = \exp\frac{v_0 x_2}{\nu} - 1 \qquad (2)$$

This equation, which gives the mean velocity in the lowermost part of the "inner layer," suggests the following general relation for the mean velocity in the inner layer:

$$\frac{v_0 U_1}{u_\tau^2} = f_1\left(\frac{v_0 x_2}{\nu}\right) \qquad (3)$$

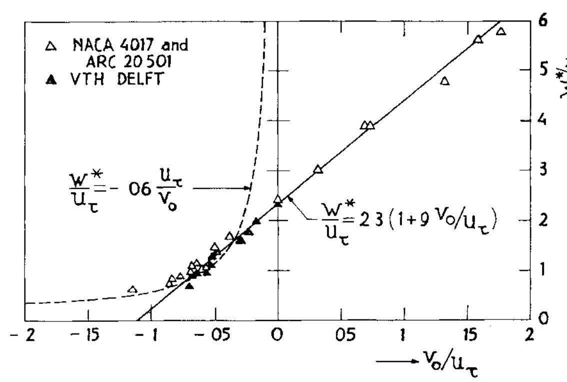

Fig 1 The logarithmic velocity scale

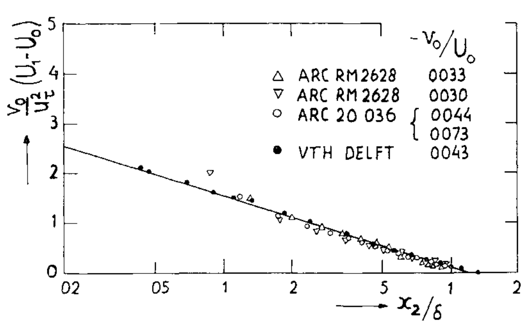

Fig 2 Mean velocity profiles of asymptotic layers

The logical choice for the related velocity defect law is

$$\frac{v_0}{u_\tau^2}(U_1 - U_0) = f_2\left(\frac{x_2}{\delta}\right) \quad (4)$$

In this equation, U_0 is the mainstream velocity For turbulent asymptotic layers in zero pressure gradient ($v_0 U_0 = -u_\tau^2$), Eq (4) appears to agree with experimental data, as Fig 2 shows

It is seen that these asymptotic layers exhibit extended regions in which their mean velocity profiles are semilogarithmic The slope of the logarithm can be represented by

$$w^* = x_2 \frac{\partial U_1}{\partial x_2} = -0\ 06 \frac{u_\tau^2}{v_0} \quad (5)$$

This relation, which probably holds for all turbulent boundary layers with moderate suction, does not conflict with the experimental data within the range $0\ 04 < -v_0/u_\tau < 0\ 10$, as is shown in Fig 1

The mean velocity profiles of Fig 2 have practically no curved "tail" at the outer edge of the layer Hence, Mickley and Smith's assertion that this tail (which is described by Coles' wake function) is unaffected by transpiration, cannot be extended to all turbulent boundary layers with suction Finally, strict similarity of mean velocity profiles according to a velocity defect law can be expected only for "equilibrium layers" in the sense defined by Clauser [4] The asymptotic layer is believed to be a proper equilibrium layer; the rather great spread in the data collected by Mickley and Smith at the larger blowing rate may partially be due to lack of equilibrium of the boundary layers concerned

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Reply by Authors to H Tennekes

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THE authors would like to thank Tennekes for his interest in their earlier paper[1] and to extend their congratulations for his contribution to the technology of sucked boundary layers However, they are concerned that Tennekes' interpretation of their paper may indicate a general misconception regarding it A semilogarithmic mean velocity profile is not necessarily "characteristic of Clauser's phenomenological description " To the contrary, Clauser's deficiency law treatment[2] is valid only for the outer portion of the boundary layer and is, therefore, more nearly analogous to Coles'[3] "Law of the Wake " For this reason, the authors refrained from drawing any conclusions with regard to the inner portion of the transpired boundary layer

The authors concur with Tennekes' belief that their concept may not be applicable to sucked boundary layers Their analysis assumed that the inner and outer portions of the boundary layers were coupled only loosely and that the maximum shear stress was representative of the stress applied to the inner face of the outer portion For sucked boundary layers, the stress applied to the outer portion is not well-defined, and the coupling assumption may be incorrect For this reason, the original article was restricted to a discussion of transpired boundary layers

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Comment on "Mathematical Analysis of Corotating Nose-Gear Shimmy Phenomenon"

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LAGRANGE'S equations offer advantages in almost all dynamics problems, but their versatility does not relieve the analyst of the burden of knowing precisely how the system functions; nor if, through design or oversight, inherent nonlinearities are omitted, can large-amplitude motions be computed even though the geometric nonlinearities (automatically contributed by the method) are retained

In Ref 1 it is suggested that the geometrically and kinematically nonlinearized equations of Lagrange be used to determine the wide-angle motion of a corotating nose gear This is not possible It is not possible because of a number of concurrent causes, any one of which, acting singly, could invalidate the equations For example, with oscillation amplitudes beyond a few degrees, the lateral wheel forces become so large that the ground contact is caused to slip Under this circumstance the equations employed in Ref 1 no longer apply Additional examples of a different nature are due to coulomb damping (particularly in the strut), hydraulic (nonviscous) damper action, and nonlinear elasticity in the tires and structure None of these nonlinearities is introduced automatically by Lagrange's equations as are the geometric nonlinearities Each must be deliberately incorporated at the beginning of the analysis The sensitivity of amplitude in a neutrally stable vibration is marked The most trifling disturbances bring about disproportionate changes in vibration amplitude Hence the omission of such factors as degrees of freedom in the air frame and dynamic imbalance make amplitude computation meaningless

Regardless of the analytic procedure adopted, the information gained is no more reliable than are the assumptions that describe the system This is illustrated in the following examples

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